Math 115

Spring 2019

Lecture 20

$? \; a^2 + b^2 = c^2 \; ?$

$y = mx + b \quad ? \quad d = rt$

Ch. 5 : Factoring

writing a Polynomial in the form of Factors

GCF : Greatest Common Factor

GCF : 10 & 15

$10 = 5 \cdot 2$

$15 = 5 \cdot 3$

$\Rightarrow$ GCF = 5

GCF : 45 & 60

$45 = 3 \cdot 3 \cdot 5$

$60 = 3 \cdot 5 \cdot 2 \cdot 2$

$\Rightarrow$ GCF = $3 \cdot 5 = 15$

GCF: $20x^2$ & $32x$

$20x^2 = 2 \cdot 2 \cdot 5 \cdot x \cdot x$

$32x = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot x$

GCF = $2 \cdot 2 \cdot x = 4x$

GCF: $18x^3y^2$ & $24xy^4$ GCF = $6xy^2$

$18x^3y^2 = 2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y$

$24xy^4 = 2 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot y \cdot y \cdot y \cdot y$

$2 \cdot 3 \quad x \quad y \cdot y$

Find the GCF

1) 50 & 75

$50 = 2 \cdot 5 \cdot 5$

$75 = 5 \cdot 5 \cdot 3$

GCF = $5 \cdot 5 = 25$

2) $28x^4$ & $98x^2$

$28x^4 = 2 \cdot 2 \cdot 7 \cdot x \cdot x \cdot x \cdot x$

$98x^2 = 2 \cdot 7 \cdot x \cdot x$

GCF = $2 \cdot 7 \cdot x \cdot x = 49x^2$

3) $12x^3$, $49y^2$

$12x^3 = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x$

$49y^2 = 7 \cdot 7 \cdot y \cdot y$

GCF = 1

Factor out the GCF (Reverse of distribution)

$15x^2 - 10x = 3 \cdot 5 \cdot x \cdot x - 2 \cdot 5 \cdot x$

$= 5x(3x - 2)$

To verify, we can distribute.

Factor out the GCF:

$20x^3 - 32x = \boxed{4x(5x^2 - 8)}$

$= 2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot x - 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot x$

Factor out the GCF:

$5x(2x+3) + 7(2x+3)$

$= (2x+3)(5x+7)$

Factor out the GCF:

$25x^3y^2 - 35x^2y^3 + 45x^2y^2$

$= 5 \cdot 5 \cdot x \cdot x \cdot x \cdot y \cdot y - 5 \cdot 7 \cdot x \cdot x \cdot y \cdot y \cdot y + 5 \cdot 3 \cdot 3 \cdot x \cdot x \cdot y \cdot y$

$= \boxed{5x^2y^2(5x - 7y + 9)}$

Factor by grouping (4 or more terms)

$2x^3 + 5x^2 + 4x + 10$

$= x^2(2x + 5) + 2(2x + 5)$

$= \boxed{(2x + 5)(x^2 + 2)}$ to verify, we can FOIL.

Factor by grouping:

$4x^3 - 7x^2 + 12x - 21$

$= x^2(4x - 7) + 3(4x - 7)$

$= \boxed{(4x - 7)(x^2 + 3)}$ verify by FOIL.

Factor by grouping:

$10x^2y + 20xy - 3x - 6$

$= 10xy(x + 2) - 3(x + 2)$

$= \boxed{(x + 2)(10xy - 3)}$

Factor by grouping

$7x^3 + 9x^2 - 35x - 45$

$= x^2(7x + 9) - 5(7x + 9)$

$= (7x + 9)(x^2 - 5)$

$10x^3(2x+5) - 15x^2(2x+5) + 10x(2x+5) - 6(2x+5)$

$= (2x+5)(10x^3 - 15x^2 + 10x - 6)$

Factor out the GCF or Factor by grouping:

1) $18x^4 - 27x^2 = 9x^2(2x^2 - 3)$

2) $5x^3 - 9x^2 + 15x - 27$

$= x^2(5x - 9) + 3(5x - 9)$

$= (5x - 9)(x^2 + 3)$

Factoring Trinomials: ax^2+bx+c

$2x^2 + 7x + 5$

$P = 10$

$S = 7$

$1, 10$

$\boxed{2, 5}$

10

$= 2x^2 + 2x + 5x + 5$

$= 2x(x+1) + 5(x+1)$

$= \boxed{(x+1)(2x+5)}$ Verify by FOIL

$3x^2 - 11x + 8$

$P = 24$

$S = -11$

$-1, -24$

$-2, -12$

$\boxed{-3, -8}$

$-4, -6$

24

$= 3x^2 - 3x - 8x + 8$

$= 3x(x-1) - 8(x-1)$

$= \boxed{(x-1)(3x-8)}$

$5x^2 + x - 4$

-20

$P = -20$

$S = 1$

$-1, 20$

$-2, 10$

$-4, 5$

$= 5x^2 + 5x - 4x - 4$

$= 5x(x + 1) - 4(x + 1)$

$= \boxed{(x + 1)(5x - 4)}$

$4x^2 + 12x + 9$

36

$P = 36$

$S = 12$

$1, 36$

$2, 18$

$3, 12$

$4, 9$

$\boxed{6, 6}$

$= 4x^2 + 6x + 6x + 9$

$= 2x(2x + 3) + 3(2x + 3)$

$= (2x + 3)(2x + 3)$

$= (2x + 3)^2$

$25x^2 - 20x + 4$

100

$P = 100$ ✓

$S = -20$ ✓

$-1, -100$

$-2, -50$

$-4, -25$

$-5, -20$

$-10, -10$

$25x^2 - 10x - 10x + 4$

$= 5x(5x - 2) - 2(5x - 2)$

$= (5x - 2)(5x - 2)$

$= \boxed{(5x - 2)^2}$

Special Factoring:

Binomials

$A^2 + B^2 \Rightarrow$ Prime

$A^2 - B^2 = (A + B)(A - B)$

$x^2 + 36 = x^2 + 6^2 \Rightarrow$ Prime

$x^2 - 36 = x^2 - 6^2 = (x + 6)(x - 6)$

Difference of two Squares

Product of Conjugates

$25x^2 + 49 = (5x)^2 + 7^2$

Sum of two squares

Prime

$25x^2 - 49 = (5x)^2 - (7)^2$

Difference of two squares

Product of conjugates

$= (5x - 7)(5x + 7)$

$81x^2 + 64y^2 = (9x)^2 + (8y)^2$

Sum of two squares

= Prime

$81x^2 - 64y^2 = (9x)^2 - (8y)^2$

Difference of two squares

$= (9x - 8y)(9x + 8y)$

Conjugates

$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

Sum of two Cubes

$x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$

$8x^3 + 27 = (2x)^3 + (3)^3 = (2x + 3)(4x^2 - 6x + 9)$

$125x^3 + 64y^3 = (5x)^3 + (4y)^3 = (5x + 4y)(25x^2 - 20xy + 16y^2)$

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

Difference of two Cubes

$x^3 - 1000 = x^3 - 10^3 = (x - 10)(x^2 + 10x + 100)$

$64x^3 - 125y^3 = (4x)^3 - (5y)^3$

$= (4x - 5y)(16x^2 + 20xy + 25y^2)$

Factor

1) x^2+49

$=x^2+7^2$

Prime

2) $x^2-121=x^2-11^2$

$=(x+11)(x-11)$

3) x^3+64

$=x^3+4^3$

$=(x+4)(x^2-4x+16)$

4) x^3-125

$=x^3-5^3$

$=(x-5)(x^2+5x+25)$

$-25x^7y^3 \quad +40x^6y^4 \quad -80x^5y^5 \quad +75x^4y^6$

D=7+3=10, C=-25 | D=6+4=10, C=40 | D=5+5=10, C=-80 | D=4+6=10, C=75

✓ D = 10

✓ LC = -25 in terms of x's.

LC = 75 in terms of y's

$$\frac{x^{\frac{7}{8}}}{x^{\frac{1}{6}}} = x^{\frac{7}{8}-\frac{1}{6}} = x^{\frac{17}{24}}$$

$$\frac{7}{8}-\frac{1}{6} = \frac{7\cdot 3 - 1\cdot 4}{24}$$

LCD = 24

$$= \frac{21-4}{24} = \frac{17}{24}$$

$$\left(6.9\times 10^{-17}\right)\cdot\left(4.8\times 10^{-13}\right)$$

$$= 33.12\times 10^{-17+(-13)} = 33.12\times 10^{-30}$$

$$= 3.312\times 10^{1}\times 10^{-30} = \boxed{3.312\times 10^{-29}}$$

$$(7x^6 - 3x^4)(7x^6 + 3x^4)(49x^{12} + 9x^8)$$

Conjugates

$$= \left[(7x^6)^2 - (3x^4)^2\right](49x^{12} + 9x^8)$$

$$= (49x^{12} - 9x^8)(49x^{12} + 9x^8)$$

Conjugates

$$= (49x^{12})^2 - (9x^8)^2$$

$$= \boxed{2401x^{24} - 81x^{16}}$$

$$(6x^4+7)(3x^4-9)$$

$$= 6x^4 \cdot 3x^4 - 6x^4 \cdot 9 + 7 \cdot 3x^4 - 7 \cdot 9$$

$$= 18x^8 - 54x^4 + 21x^4 - 63$$

$$= \boxed{18x^8 - 33x^4 - 63}$$

Trinomial
D = 8
L.C. = 18
Const = −63

$$\frac{x^4+13x^2+36}{x^2+x-12}$$

$x^2 \boxed{x^2} = x^4$

$x^2 \boxed{-x} = -x^3$

$x^2 \boxed{\quad} = 26x^2$

$$x^2 - x + 26 + \frac{-38x+348}{x^2+x-12}$$

$$\begin{array}{r|l} & x^2-x+26 \\ \hline x^2+x-12 & x^4+0x^3+13x^2+0x+36 \\ & -(x^4+x^3-12x^2\ \ \ ) \\ \hline & -x^3+25x^2+0x+36 \\ & -(-x^3-x^2+12x\ ) \\ \hline & 26x^2-12x+36 \\ & -(26x^2+26x-312) \\ \hline & -38x+348 \end{array}$$

$$\frac{36x^8 - 24x^4 + 12x^2}{-12x^2}$$

$$= \frac{36x^8}{-12x^2} - \frac{24x^4}{-12x^2} + \frac{12x^2}{-12x^2}$$

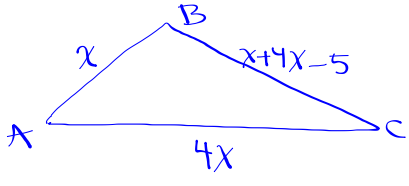
$$= \boxed{-3x^6 + 2x^2 - 1}$$

Trinomial
D = 6
LC = −3
Const = −1

In triangle ABC, one side is 4 times another side.

The third side is 5 ft shorter than the sum of first two sides.

Find all three side if p = 31 ft.

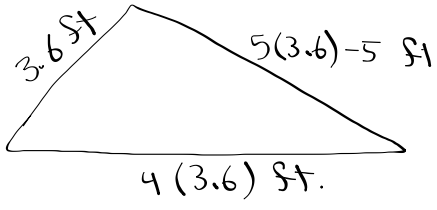


$P = 31$

$x + 4x + 5x - 5 = 31$

$10x - 5 = 31$

$10x = 36$ $\quad \boxed{x = 3.6}$



John has \$1.35 in nickels & Dimes.
He has 20 coins.
How many of each?

N → Nickels
D → Dimes

$$\begin{cases} N + D = 20 \\ 5N + 10D = 135 \text{¢} \end{cases} \quad \div 5$$

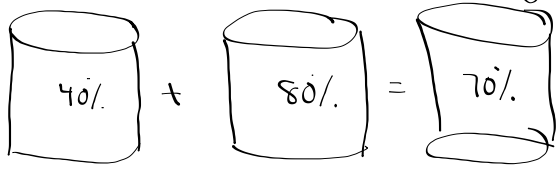
$$\begin{cases} N + D = 20 \\ N + 2D = 27 \end{cases} \quad -1$$

$$\begin{cases} -N - D = -20 \\ N + 2D = 27 \end{cases}$$

$$D = 7$$

7 Dimes
&
13 Nickels

Lisa needs 100 liters of 70% acid Solution.
She has unlimited supply of 40% acid and
80% acid Solutions. How many liters of each?



$$\begin{cases} x + y = 100 \\ .4x + .8y = .7(100) \end{cases} \quad 10$$

$$\Rightarrow \begin{cases} x + y = 100 \\ 4x + 8y = 700 \end{cases} \quad \div 4$$

$$\begin{cases} x + y = 100 \\ x + 2y = 175 \end{cases} \quad -1$$

$$y = 75$$

75L of 80% Acid
25L of 40% Acid.

Simple interest:

$I = P \cdot r \cdot t$

- I: deposit
- r: rate
- t: time

Moe deposited $2500 for 18 months at Simple interest annual rate of 4%.

How much interest?

$I = P \cdot r \cdot t$

18 months → years

$= 2500 (.04)(1.5) \Rightarrow \boxed{I = \$150}$